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Active Vibration Control of Cantilever Beam System Using Particle Swarm Optimization

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Abstract— This paper presents an application of deploying an adaptive state-space feedback controller for vibration suppression in a flexible cantilever beam system using single and multiple control actuators and deflection sensors optimized with an On-line Particle Swam Optimization (PSO). The PSO is adopted for the optimization problem because of its proved simplicity and performance in different linear and nonlinear applications. Neither expensive computations nor specialized methods are needed. The behavior of the complex nonlinear beam system makes it difficult to locate a global minimum. This nonlinear vibration problem is equivalently changed to the problem of function optimization, which is tackled by using PSO, where a population-based stochastic optimization technique inspired by the social behavior of bird flocking is used, The approach is validated using numerical simulations; and the results confirm the effectiveness of the PSO approach and its ability to highly improve the vibration cancellation performance of the flexible beam system.

Keywords— Active vibration control, Cantilever beam, Control actuator, Deflection sensors, Flexible beam structure, Optimization, Particle swam optimization algorithm.

I. INTRODUCTION

Engineering structures/plants with flexible parts are usually associated with inherent vibration properties. The nature and extent of this vibration depend on the type of the structure and its mountings. These vibration problems become more prominent when the structure is placed on a relatively flexible base or then there is a considerable amount of movement within the plant. Vibration control of these engineering structures is essential for their proper operation and long life cycle. A considerable amount of research work has been done in this area [1]-[4].

This paper presents an investigation into simulation and vibration control of a cantilever beam system; and it presents an effective vibration cancellation of the structure using single or multiple control actuators and observation sensors. A cantilever beam system is considered because it is used in a number of flexible structures such as aircraft wings and space structures. Due to the distributed nature of the governing equation describing structural dynamics, control of flexible structures involves a very complex process [5]. It is important initially to recognize the flexible nature of the beam, construct a mathematical model for the system and account for interactions with disturbances. The commonly used method, for numerically solving the systems governing equation, is based on finite elements (FE) and finite difference (FD). The computational complexity and consequent software coding is a major disadvantage of this technique, especially in real time system applications [6]. As FD method is generally found to be more appropriate, it is used in this paper. A reduced order model with the first few dominant modes is developed to implement the modal controller. For the implementation of the modal controller, a Kalman-Bucy filter state observer is used to estimate the state vector, which is not measured through [7], [8]. Several interesting control application areas, which adopt PSO algorithm, have recently been proposed by researchers in automatic generation control tuning [9], design of controllers [10], [11], adaptive inverse control [12], [13], predictive control [14], [9], PI and PID controllers [15] and ultrasonic motor control [16].

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Currently, PSO is a well-known effective approach for solving complex optimization problems [17], [18].

These control algorithms can provide adequate control management for wide classes of problems related to active vibration suppression though they are sensitive to structure characteristics and require an exact mathematical model of a structure even for the collocated system. Because they are sensitive to operating conditions, it is difficult to adjust controller gains. The most commonly used control algorithms are "classical" control algorithms such as direct proportional feedback, constant gain velocity feedback (CGVF) and constant amplitude velocity feedback (CAVF) control. Optimal control algorithms are also used (LQR and LQG) for active vibration suppression [19]. An alternative is the use of intelligent control algorithms based on soft computing schemes such as fuzzy logic control (FLC) algorithms [20] and active controller based PSO [21]. Therefore, FLC method has been applied widely for active vibration control of flexible structures [22].

In this paper, PSO is adopted for the vibration cancellation problem because of its proved simplicity and performance in different applications. Neither expensive computations nor specialized methods are needed. However, this research paper found that PSO outperforms random search throughout and at the end of the search process as it shows a better convergence behavior and over-fitting avoidance. This result indicates that PSO can work as any-time method for the vibration suppression problem of flexile structures. In addition, PSO based-state feedback controller, as compared to the general state feedback controller, performs extremely well in high dimensional optimization problems. Consequently, it is used to find the proper and effective state feedback parameters and remove the tedious and repetitive trial and error process of using traditional techniques [23]. Evolutionary optimization algorithms are used to design the controller for vibration cancellation. The proposed method operates better in the aspect of designing the controller since it provides ample opportunities for designers to choose the most appropriate point based upon design criteria [24]. The simulation results of the beam system behavior and the effectiveness of the controller on vibration reduction using single or multiple control actuators and sensors are important features of this paper.

II. CANTILEVER BEAM SYSTEM SIMULATION

The motion of vibrating structures can be described by a partial differential equation. For a fixed-free cantilever beam structure (the data for the beam is presented in Appendix A), the time response of the system in lateral motion can be described by the fourth order of partial differential equation (PDE) [25]:

$$EI\frac{\partial^4 y(\alpha,t)}{\partial\alpha^4}+m\frac{\partial^2 y(\alpha,t)}{\partial t^2}=f_d(\alpha,t)+f_c(\alpha,t) \tag{1}$$

where α is the distance along the beam from the fixed end; $y(\alpha,t)$ is the displacement of the beam at point α and at time *t*, $f_d(\alpha,t)$ and $f_c(\alpha,t)$ are the disturbing and controlling signals acting on the beam at point α and time *t* respectively; *m* is the beam mass per unit length; and *EI* is the flexural rigidity of the beam. The corresponding boundary conditions at the fixed and free ends of a cantilever of length *l* are:

$y(0,t)=0$ and $\frac{\partial y(0,t)}{\partial\alpha}=0$ for the fixed end, $\frac{\partial^3 y(l,t)}{\partial\alpha^3}=0$ and $\frac{\partial^2 y(l,t)}{\partial t^2}=0$ for the free end.

To construct a suitable simulation platform for testing and verifying control mechanisms, the central FD is the most accurate and deployed form in simulation environment. After the discretization of (1) and subsequent rearrangement, it displaces various beam elements. Using matrix form for the fixed end yields:

$$Y_{j+1} = -Y_{j-1} - \lambda^2 S Y_j + \frac{(\Delta t)^2}{[\rho A]} f(\alpha, t) \tag{2}$$

where $\lambda^2 = \frac{\mu_n{}^2 \Delta t^2}{\Delta \alpha^4}$ and $\mu_n{}^2 = \frac{EI}{\rho A}$ and

$$Y_{j+1} = \begin{bmatrix} y_{1,j+1} \\ y_{2,j+1} \\ \vdots \\ y_{n,j+1} \end{bmatrix}, \; Y_j = \begin{bmatrix} y_{1,j} \\ y_{2,j} \\ \vdots \\ y_{n,j} \end{bmatrix}, \; Y_{j-1} = \begin{bmatrix} y_{1,j-1} \\ y_{2,j-1} \\ \vdots \\ y_{n,j-1} \end{bmatrix}$$

and S is a penta-diagonal matrix given by:

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cdots & \cdots & 0 \\ -4 & b & -4 & 1 & 0 & 0 & \cdots & \cdots & 0 \\ 1 & -4 & b & -4 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & -4 & b & -4 & 1 & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & 1 & -4 & b & -4 & 1 \\ \cdots & \cdots & \cdots & \cdots & 0 & 1 & -4 & a & -2 \\ \cdots & \cdots & \cdots & \cdots & 0 & 0 & 2 & -4 & c \end{bmatrix} \tag{3}$$

where $a = 5 - \frac{2}{\lambda^2}$, $b = 6 - \frac{2}{\lambda^2}$, $c = 2 - \frac{2}{\lambda^2}$ and $\mu^2 = \frac{EI}{m}$.

The necessary and sufficient condition of stability for this method to give a convergent solution is satisfied if $0 < \lambda^2 \leq 0.25$ [7]. The schematic diagram of the beam system for 20 stations (sections) is illustrated in Fig. 1. The characteristic equation [25] for the fixed-free beam is:

$$\cos(\beta_i l)\cosh(\beta_i l) = -1 \text{ for } i = 1, 2, ..., n \tag{4}$$

where l is the beam length; $\beta_i l$ are the roots of the characteristic equations; n is the number of the considered modes; and $\beta_i{}^4 = \frac{\omega_i{}^2}{\mu^2}$, ω_i are the resonant frequencies; The natural frequencies of the first five modes are 1.14, 7.12, 19.93, 39.05 and 64.56Hz respectively.

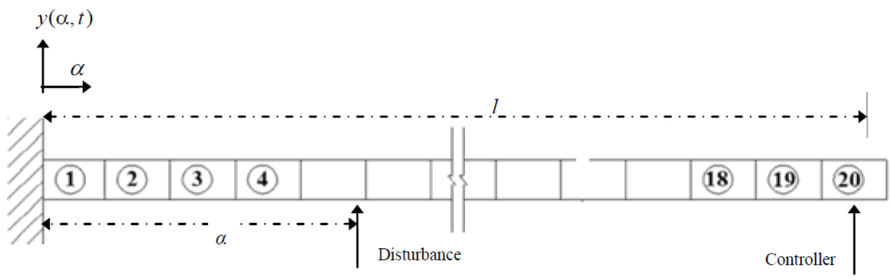


Fig. 1. Schematic diagram of the cantilever beam

The deformation of the beam is measured by Hall Effect position sensors (observation points). The measurements are utilized to generate the control force using some design philosophy. Cancellation forces are implemented via control electromagnetic actuators to suppress vibrations at control points. Clearly, there are many configurations arising from possible variations in the number of observation points, control actuators and control points together with possible various locations for the sensors and actuators. It is considered to have different number of sensors and control actuators. Moreover, the number of excited modes depends on the disturbance location; for example, if disturbance is located at a modal node, this mode will not be excited. In a similar manner, if the observation point and/or the control actuator are placed at a modal node, this mode will be unobservable and/or uncontrollable [26]. In addition, the position of control components (sensors and actuators) should be chosen carefully for effective control based on the beams' nodes and antinodes as well as the optimization technique as indicated in Appendix A. A different number of sensors and actuators is utilized for observing or controlling the vibration of a different number of modes. In our work, a modal technique is utilized to control the beam's modes using both collocated and/or non-collocated sensor/actuator pairs.

III. PSO Method

The PSO algorithm was originally proposed by Kennedy and Eberhart in 1995 [27].The PSO algorithm is an evolutionary computational technique, but it differs from other well-known evolutionary computational algorithms such as genetic algorithms. Although population is used for searching the search space, there are no operators applied on the population. Instead, in PSO, population dynamics simulate a 'bird flock's' behavior, where social sharing of information takes place; and individuals can profit from the discoveries and previous experiences of all the other companions during their search for food. Thus, each companion, called particle, in the population, which is called swarm, is assumed to 'fly' over the search space in order to find promising regions of the landscape. Optimization methods based on swarm intelligence are called behaviorally inspired algorithms as opposed to genetic algorithms, which are called evolution-based procedures [23].

In the context of multivariable optimization, the swarm is assumed to be of specified or fixed size with each particle located initially at random locations in the multidimensional design space. Each particle is assumed to have two characteristics: a position and a velocity. In addition, it wanders around in the design space; and remembers the best position (in terms of the food source or objective function value) it has discovered. Particles communicate information or good positions to each other; and adjust their individual positions and velocities based on the information received on good positions. Thus, the PSO model simulates a random search in the design space for the maximum value of the objective function. As such, gradually over much iteration, birds go to the target (or maximum/minimum of the objective function) [27].

Let x and v denote a particle position and its corresponding flight velocity in a search space, respectively. Therefore, the i^{th} particle is represented as $x^i = (x^{i1}, x^{i2}, ..., x^{id})$ in the d-dimensional search space. The best remembered of the i^{th} individual particle position is recorded and represented as $pbest^i = (pbest^{i1}, pbest^{i2}, ..., pbest^{id})$. The index of the best remembered swarm position among all the particles in the group is represented by the $gbest = (gbest^1, gbest^2, ..., gbest^{d)}$. The flight velocity of particle i is represented as $v^i = (v^{i1}, v^{i2}, ..., v^{id})$. The modified velocity and position of each particle can be calculated using the current velocity and the distance from $pbest^i$ to $gbest$ as presented in the flow chart shown in Fig. 2 where N_{od}, N_{oi}, and N_{op} are the number of dimensions, the maximum iteration number, and the number of particles respectively [11], [16]. In addition, w is adopted as the inertia weight factor; c_1 and c_2 are acceleration constants.

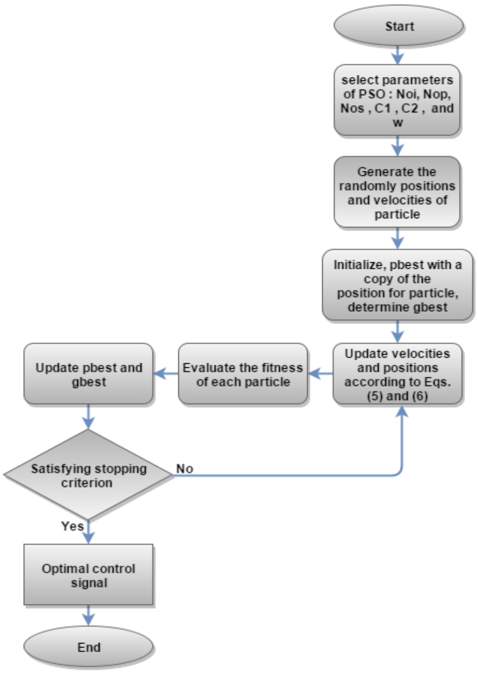


Fig. 2. Block diagram of PSO algorithm

The modified velocity and position of each particle can be calculated using the current velocity and the distance from $pbest^{id}$ to $gbest^d$ as presented in the following formulas [10], [27]:

$$v_{k+1}^{id} = wv_k^{id} + c_1 r_1 (pbest_k^{id} - x_k^{id}) + c_2 r_2 (gbest^d - x_k^{id}),\ i=1, 2,..., n \text{ and } d=1, 2,\ldots, m \quad (5)$$

$$x_{k+1}^{id} = x_k^{id} + v_{k+1}^{id} \quad (6)$$

where c_1 and $c_2 \geq 0$, n are the number of particles in a group; m is the number of members in a particle; r_1 and r_2 are two random numbers between 0 and 1; x_k^{id} and v_k^{id} are the velocity and current of particle i in the d^{th}-dimensional search space at iteration k, respectively.

In general, PSO shares many similarities with evolutionary computational techniques. The main difference between the PSO and other approaches is that PSO does not have operators. Particles update themselves with the internal velocity; they also have a memory that is important to the algorithm. In addition, PSO is easy to implement as it has few parameters to adjust. Furthermore, PSO is computationally inexpensive since its memory and speed requirements are low [22].

IV. Simulation Results and Performance Evaluation

To investigate the effectiveness of the PSO-based controller on the performance of a cantilever beam system, the FD approximation of the beam system using 20 stations is utilized. The integration step-size of the FD simulation is chosen to be 0.33ms, which is sufficient to cover the dominant resonance modes of the beam; this has been demonstrated to yield a good level of accuracy. All simulation results are carried out when a step disturbance force of 0.1N is assumed to be applied at a location 0.65*l* from the fixed end of the beam. The three dimensional description of the vibration of the uncontrolled beam along its length is shown in Fig. 3 where the beam deflection is zero at the clamped end. It increases with the distance α from the fixed end, and reaches a maximum distance at the free end. This result is as expected; and it confirms the validity of the FD simulation in representing the behaviour of the cantilever beam.

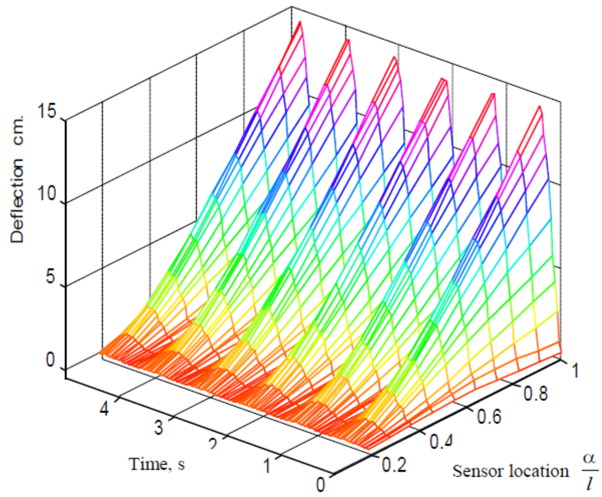
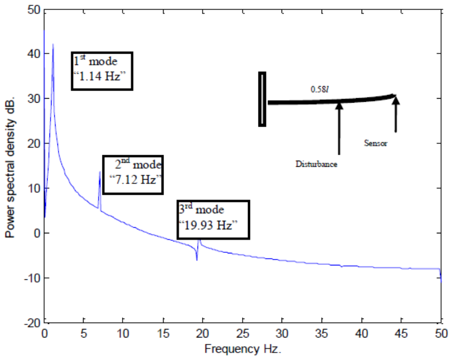
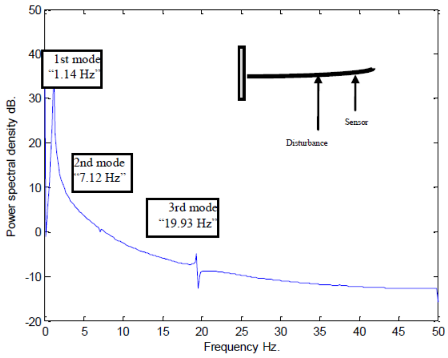


Fig. 3. Time response of the un-controlled cantilever beam along its length

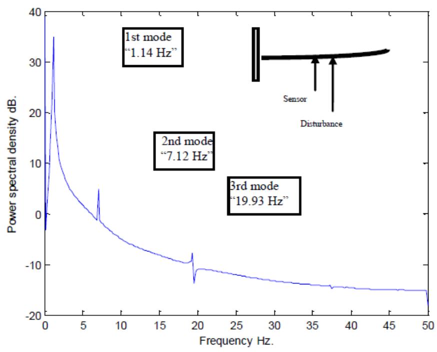
Before presenting control simulation results, the most dominant modes, which do exist in our beam system, need to be illustrated. It is assumed that the beam is disturbed by a unit step force of 0.1N by an actuator placed at the end of the beam. An indication of how many modes exist can be attained by studying the power density spectrum of the beam's time response to disturbances (i.e. a step). Such a spectrum density plot is as shown in Fig. 4 at different observation points along the beam, where sensing characteristics for the first three modes seem to be satisfactory. However, the beam under consideration in this study has relatively low sensitivity for the higher (residual) modes. It can be observed that the first three modes are dominant. It should be noted that, the power density of some nodes has disappeared or equalled zero; this is because the deflection sensor is located at the beam nodes of such modes. The results clearly show that the effect of the location of the sensor is as important as the location of the control actuator. This is due to the fact that when the sensor is located, for example, at a node of one (or more) of the dominant modes, the behaviour of this mode will be unobservable (or undetectable) and then uncontrollable. The nodes and the antinodes of the cantilever beam are presented in Appendix A.



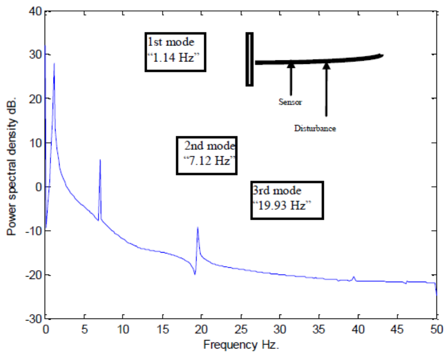
a) Sensor location at l from clamped end



b) Sensor location at $0.8l$ from clamped end



c) Sensor location at $0.6l$ from clamped end



d) Sensor location at $0.4l$ from clamped end

Fig. 4. Frequency response of the uncontrolled beam deflections at different locations along its length

In order to demonstrate the effectiveness of the PSO in vibration suppression, the evolution procedure of PSO algorithms which was shown in Fig. 2 has been considered. The structure of the controller based-PSO algorithms is shown in Fig. 5. Moreover, the frequency and time responses are chosen as the performance indices to be obtained. Since computational time is one of the important factors to be considered in an optimization process, investigations on the number of individuals/particles (N_{op}) were carried out by varying those numbers from 10 to 200. Fewer individuals/particles resulted in high values of errors but faster computational time, while a high number of individuals/particles resulted in smaller values of the mean error with a very slow execution time. In order to get compromise values between the mean error and computational time, the best number of individuals/particles was found to be 50 for all algorithms The other parameters considered for PSO algorithm are c_1= 2, and c_2= 2. Moreover, the number of dimensions (N_{od}) is five; the maximum iteration number (N_{oi}) is 20. They are used for checking the termination criterion in this algorithm. Consequently, the inertia weight factor (w) is selected according to the following equation:

$$w = 0.9 - 0.7 * (i / N_{oi}) \text{ where } i \text{ is the } i^{th} \text{ iteration} \quad (7)$$

The decreasing of w through the search process, called adaptive inertia weight, is a process similar to that of simulated annealing in which temperature is decreased exponentially, allowing global and local search [16]. As in most search algorithms, a cost function in PSO is needed to evaluate the aptitude of candidate solutions. Generally, the definition of a cost function depends on the problem at hand. However, it should generally reflect the proximity of the solutions to the optima. The cost function adopted in this work is selected to be based on the sum squared error between deflection at observation points and reference values a according to a reference model. It should be noted here that the cost function in this work, when using two control actuators system, is chosen to have the following form:

$$ft(p) = e^{k\times\sqrt{error_{o1}{}^2(\alpha)+error_{o2}{}^2(\alpha)}} \tag{8}$$

where $error_{o1}(\alpha)$ is the error between reference values and deflection at the beam free end; and $error_{o2}(\alpha)$ is the error between reference values and the deflection measured at the selected locations along the beam length.

In many practical cases, only a limited number of effective or dominant modes is excited. The objective of active damping is to suppress the vibration of such dominant modes. The controller design methodology is proposed to determine the feedback gains and compute the suitable control signal applied to a control actuator which is placed in a suitable location at the flexible structure. Generating the control signal is based upon the response measured by velocity detection sensors. Here, the control problem to be resolved is to add damping to the un-damped structure system by feeding back velocity. The complete information about the linear state feedback technique is presented in [1]. More specifically, the PSO optimizes the linear feedback gains based on minimizing the steady state error between a reference model and the beam system outputs at a specific location along the beam through a cost function. The state feedback control system is illustrated in Fig. 5. The first three modes in the system model are defined as:

$$u_c(t) = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{14} & f_{15} & f_{16} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} \tag{9}$$

where $f_{11}, f_{12},..., f_{16}$ are the position and velocity feedback gains for three modes design; and $x_1(t), x_2(t),..., x_6(t)$ are the position and velocity system states. To simplify the PSO duties, the optimal designed control law is chosen to depend on modal velocity states. The feedback gains f_{11}, f_{12} and f_{13} are all assigned to zeros. In the beam system model, the dynamical equation of this observer in the vibration application is given by [3]:

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{A}\hat{\boldsymbol{x}}(t) + \boldsymbol{B}\boldsymbol{u}_c(t) + \boldsymbol{D}\boldsymbol{u}_d(t) + \boldsymbol{L}(\boldsymbol{y}_s(t) - \boldsymbol{y}_m(t)) \tag{10}$$

$$\boldsymbol{y}_m(t) = \boldsymbol{C}\hat{\boldsymbol{x}}(t) \tag{11}$$

where *L* is the $2n\times p$ observer gain matrix; *p* is the number of deflection sensor; *n* is the number of modes considered in the system; $y_s(t)$ is the system output; $y_m(t)$ is the observer output (-dim *p*), *uc(t)* and *ud(t)* are the control and disturbance forces acting on the structure respectively; *A* is the system matrix, *B, D,* C and *F* are respectively the control, disturbance, output displacement and the state feedback gain vectors of the beam structure.

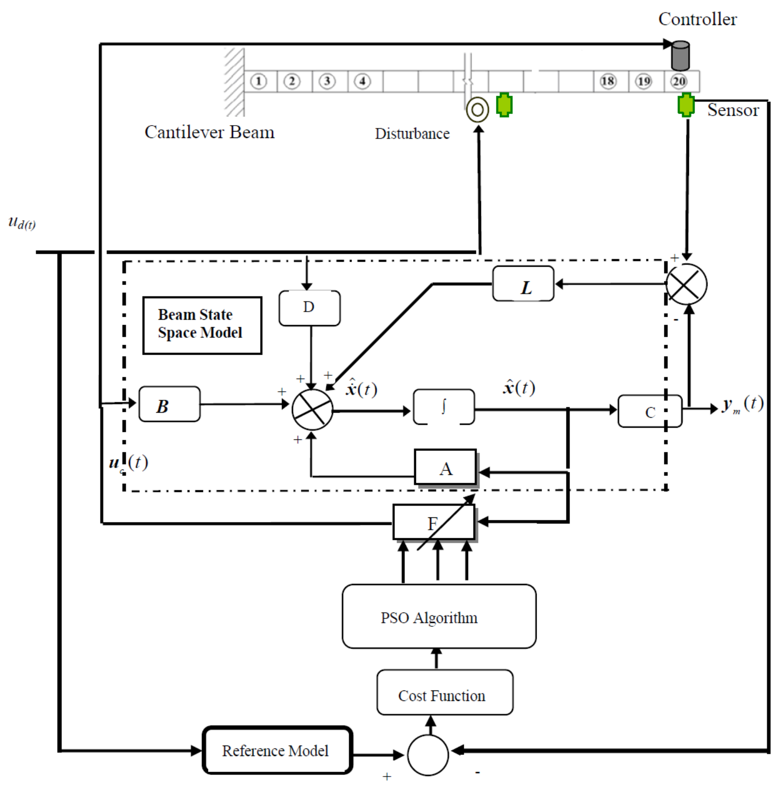
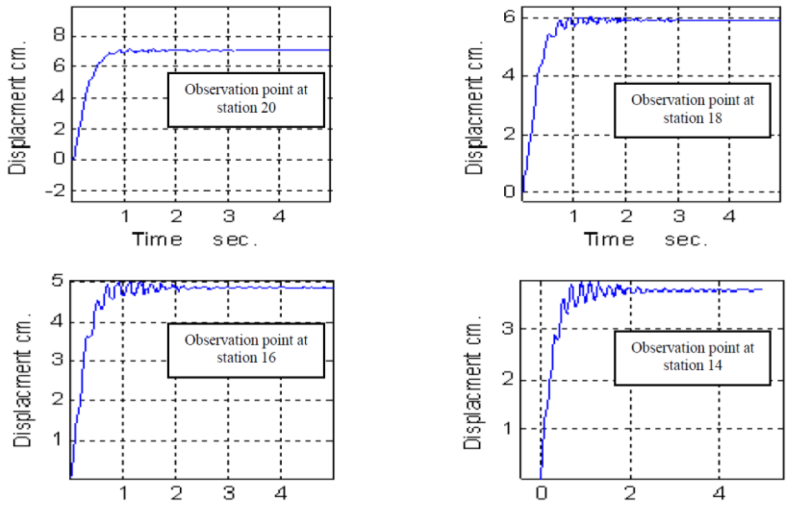


Fig. 5. Structure of state feedback control system

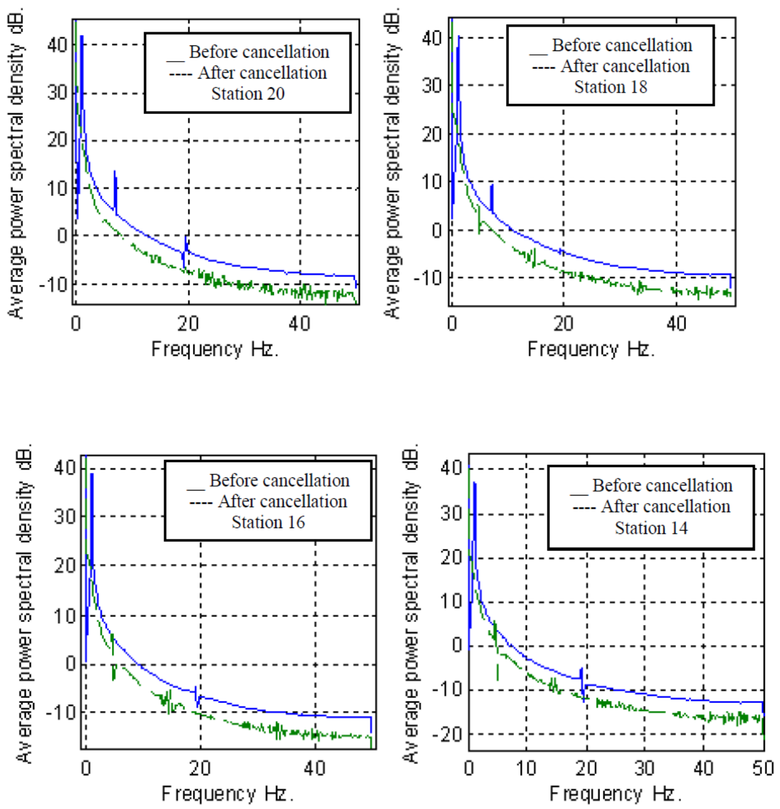
Before starting the vibration suppression procedure, the placement of sensors and actuators has to be carefully designed; a significant factor to consider is whether they collocated or not with the nodes and anti-nodes of the resonant modes. However, the optimal positioning problem, which effectively controls vibration with less control effort, becomes very important. In this paper, the location of sensors and control actuators is determined, so that the sum of the absolute values of mode shapes of all the considered modes at the same location may become maximal. Mode shapes and their contributions at each point along the beam length are shown in Appendix A.

A. Simulation Results with Single Control Actuators

In this paper, control was implemented via a single controller placed at the free end because this location is not near a node (minimum deflection) of the considered modes. Using the on-line optimized feedback gains in the control system design, where the convergence of the error value and gain parameters corresponding to the best of all particles, yields the overall results in time and frequency domains that are obtained at different observation points as shown in Fig. 6. The results in this case demonstrated a clear reduction in vibration levels for the dominant modes. These results clearly show that the controller was effective for local minimisation, but it was not enough for obtaining global cancelation through the beam length. To improve system performance, multiple controllers were adopted at some suitable locations along the beam length in demonstrated at Section *B.*



a



b

Fig. 6. Overall results in time and frequency domains at different observation points: a) Controlled beam time responses at different observation points, b) Corresponding power spectral density at different locations along the beam length

Figs. 7 and 8 show time domain results of the on-line PSO-based controller along the beam length using a unit step disturbance signal with different magnitudes when the controller is located at the free end location. It is noted in Figs. 7 and 8 that vibration at the first three modes is attenuated significantly when the disturbance force equals 0.1N, or when its magnitude is changed to 0.05N after 2.5s from the time of simulation. It can be concluded that the controller works well for the beam system, where good damping is observed.

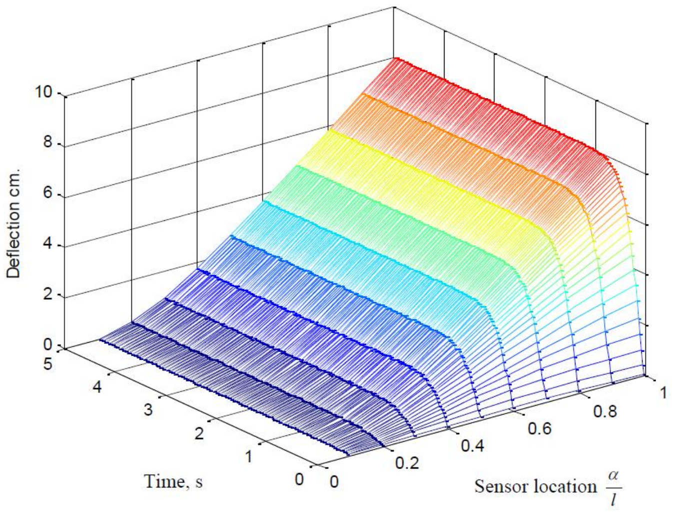


Fig. 7. Time response of the controlled cantilever beam along its length when the disturbance value equals 0.1N

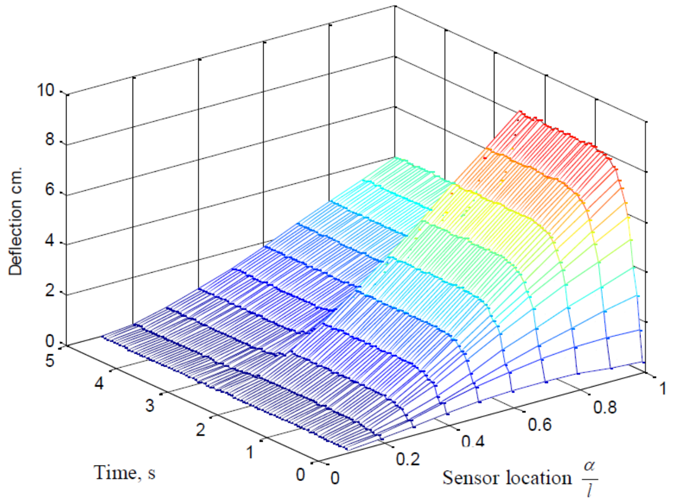


Fig. 8. Time response of the controlled beam along its length when the disturbance value is changed to 0.05N after 2.5s from stating time

Fig. 9 demonstrates the average power spectral density throughout the length of the beam before and after cancellation for a collocated case. The mean attenuation over all the considered modes is found to be 11.11dB. It can be concluded that the modal controller can be implemented successfully for suppressing the vibration of flexible structures.

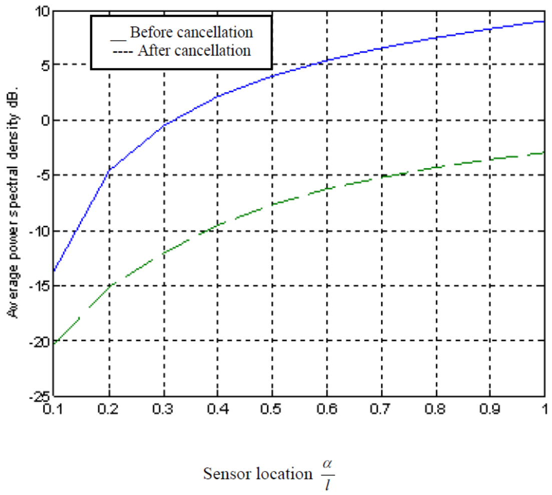


Fig. 9. Average power spectral density before and after cancellation over the beam length

B. Simulation Results with Multiple Control Actuators and Deflection Sensors

To assess the behaviour of using two controllers upon vibration cancellation over the entire beam length, controllers and the detection sensor are located at stations 20 and 13 for collocated cases. The controllers are conducted on from the starting time of simulation; the overall results are shown in Fig. 10 as measured at different locations along the beam structure. These results demonstrate and confirm the capability of the suggested procedure and control strategy used to efficiently control vibration in the structure, where global minimization along the beam is observed.

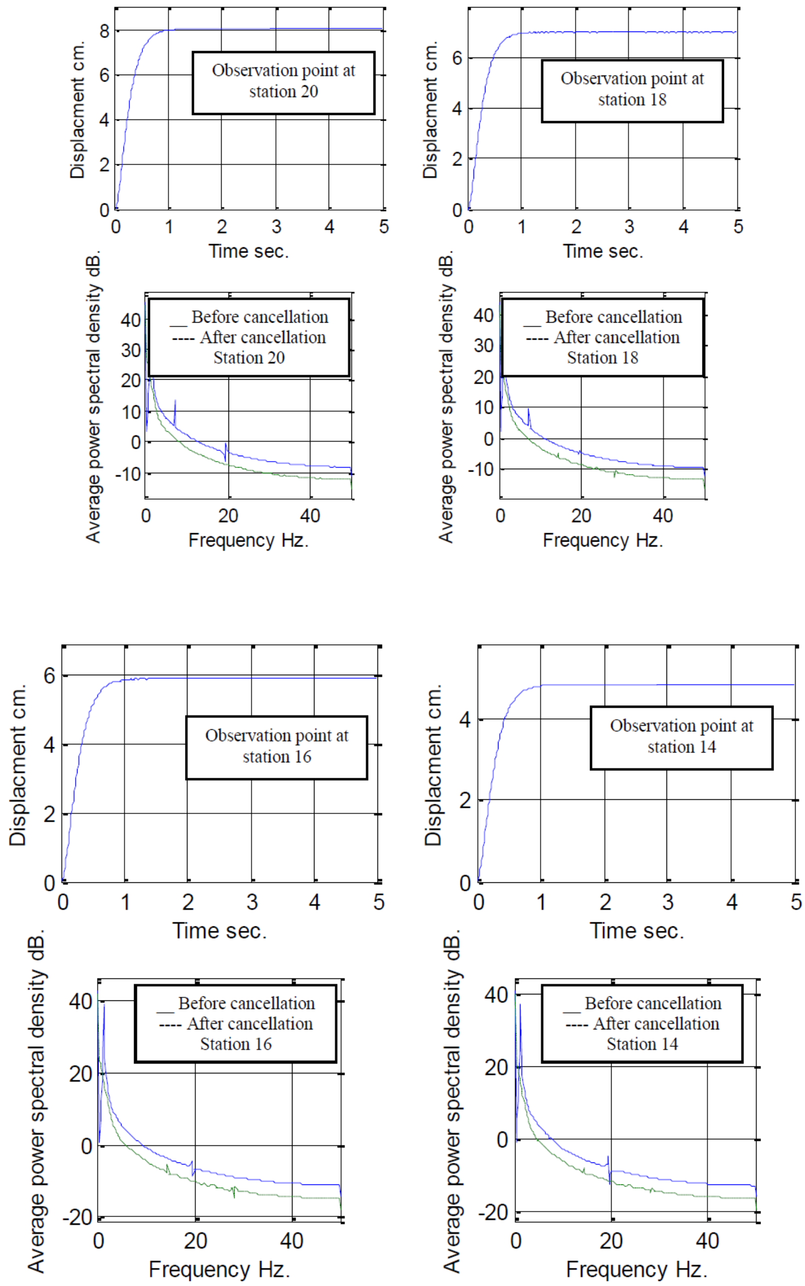


Fig. 10. Controlled structure responses within time and frequency domains measured at different locations along the beam

The three dimensional description of the vibration of the controlled beam along its length is illustrated in Fig. 11 where two conductors are deployed with two deflection sensors. It is noticed that the beam deflection is zero at the clamped end, but it reaches its maximum gradually at the free end. This result is as expected; and it confirms the validity of the FD simulation in representing the behaviour of the controlled cantilever beam.

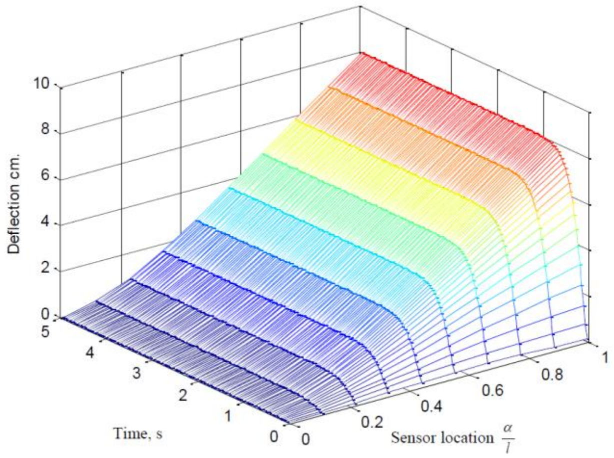


Fig. 11. Response along its length after cancellation for a collocated case with two controllers and two sensors

Fig. 12 demonstrates the mean power spectral density throughout the length of the beam before and after cancellation for a collocated case. Average attenuation over all the considered modes is found to be 12.91dB. It should be noted that a better performance has been obtained for the two controllers' scheme, where attenuation is found to be higher than that obtained for a single controller case. It can be concluded that the adopting of multiple controllers can be implemented successfully for suppressing the vibration of flexible structures.

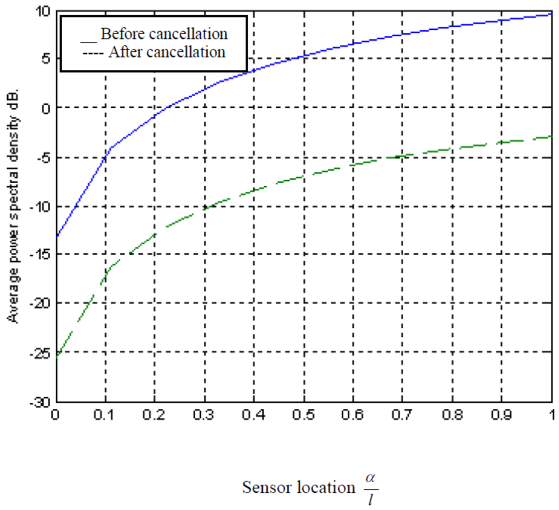


Fig. 12. Average power spectral density before and after cancellation for the collocated case with two controllers and two deflection sensors

C. Simulation Results with Multiple Control Actuators and Single Deflection Sensor

To investigate the effectiveness of the two controllers with a single deflection sensor on the performance of a cantilever beam system, the FD approximation of the beam system using 20 stations with a single sensor located at free end is utilized. The set of results describe the controlled beam behaviour when the feedback gains are online tuned by using the PSO algorithm. In the present case, simulation is carried out for a 5s when one of the controller actuator s and observation point are assumed to be located at the free end of the beam; and the other one is located at station 13. Fig. 13 shows the time response of the controlled beam system, where reasonable reduction in the vibration level can be observed along the beam length. This is further evidenced in the average corresponding frequency-domain description in Fig. 14, where a clear reduction in vibration levels for the dominant modes can be observed. These results clearly show that controllers were less effective with only one deflection sensor than when two sensors are deployed. The mean power spectral in this case is found to be equal 11.55dB.

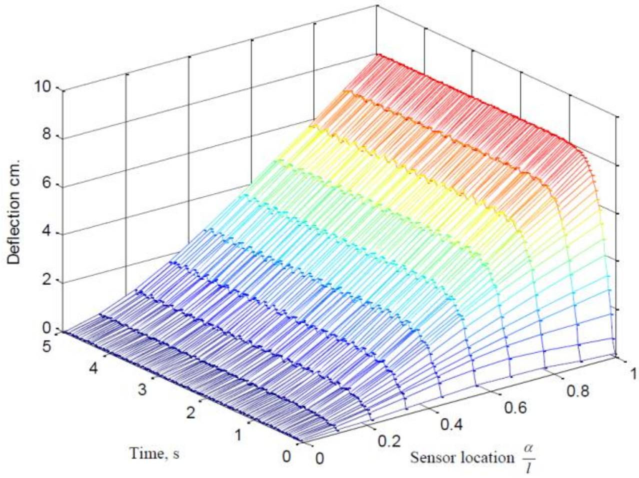


Fig.13. Beam response after cancellation with two controllers and single deflection sensor along the beam length

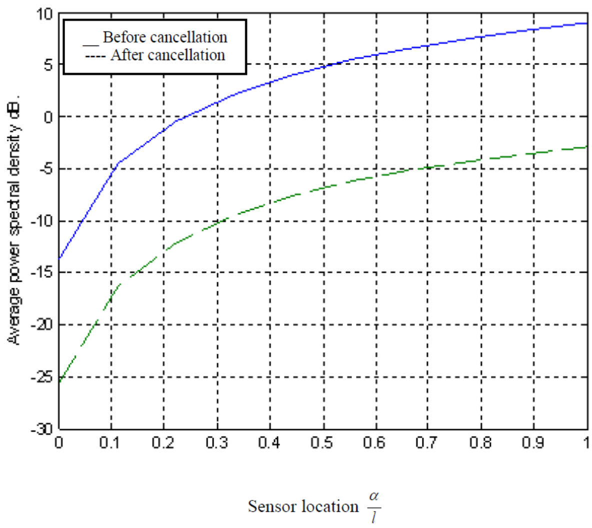


Fig. 14. Average power spectral density before and after cancellation when two controllers and a single senor are deployed

Actually, one of the main problems faced when using this kind of solution is its enormous computational demands because the PSO should work during all the period. This makes the execution time quite high. For applying such on-line GA controllers on real-time applications more than one processor needs to be considered. The adaptation of parallel computing techniques to achieve real-time performances would be useful; such implementation is left for future work. This problem can be minimized by off-line optimization of the feedback gain parameters using the PSO technique. This technique is considered as an initial value in the PSO algorithm when it is presented in the on-line control process system. This indicates that the PSO is converged quickly to global minimum. It captures, for each iteration, the control signal needed with a minimum execution time.

V. CONCLUSIONS

In this paper, an approach to design PSO-optimizer based state feedback controller was presented for solving the vibration control problem. The PSO was found to be an effective approach for solving this problem because it is relatively easy to comprehend and involves simple function evaluations. Extensive simulations have been conducted and presented in this work which evaluates the ability of the PSO solution to optimize the parameters of the state feedback controller. Simulations were conducted using off-line and on-line PSO-based controllers as an adaptive parameters tuner. It can be concluded that the performance of the controller improves when multiple controllers and multiple deflection sensors use single collated controller/sensor pairs. It was found that the PSO based state feedback controller provides an effective solution for the vibration control system optimisation problem. About 12.5dB cancellation is obtained in the vibration level when two controllers with two deflection sensors are employed. 11.11dB vibration level reduction is obtained when a single collocated control actuator and sensor are considered. 11.75dB vibration cancellation is gained when simulations are conducted using two control actuators with a single deflection sensor. The results confirmed the effectiveness of the state controller; and they illustrated that the PSO algorithm has the search capabilities necessary to find the vibration problem optimal solution.

APPENDIX

Beam data

A.1. Cantilever Beam Parameters

The data about the beam used in the research described in this paper is as follows:

Beam shape: Thin rectangular solid beam

Beam material: Aluminium

Beam length, l: 0.8m

Cross section area, A: 2.1894×10^{-5} m^2

Mass per unit length, ρA: 0.05933274 Kg/m

Shear, EI: 0.100105598 $N.m^2$

Beam constant, $\mu^2 = \dfrac{EI}{\rho A}$: 1.687189872 $m^4.s^{-2}$

A.2. Cantilever Beam Nodes and Antinodes

Tables A1 and A2 show the location of the cantilever beam's nodes and anti-nodes as measured from the clamped end.

TABLE A1
LOCATIONS OF NODES ALONG THE CANTILEVER BEAM

Mode	Node Distance				
1	0	__	__	__	__
2	0	0.78l	__	__	__
3	0	0.51l	0.87l	__	__
4	0	0.36l	0.64l	0.90l	__

TABLE A2
LOCATIONS OF ANTI-NODES ALONG THE CANTILEVER BEAM

Mode	Anti-Node Distance				
1	l	__	__	__	__
2	0.47l	l	__	__	__
3	0.30l	0.69l	l	__	__
4	0.21l	0.51l	0.78l	l	__

A.3. Cantilever Beam Mode Shapes

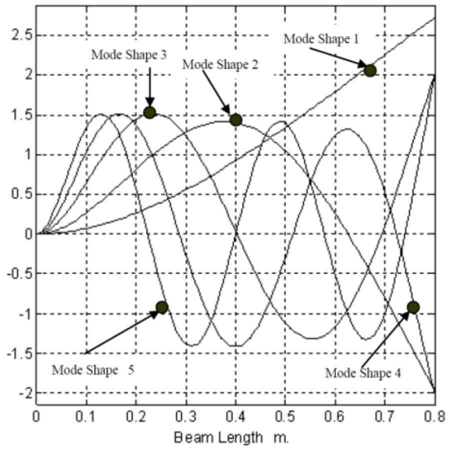


Fig. A1. Mode shapes and contribution throughout the length of a cantilever beam structure

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